الإسم = حسام الدين

اللقب = همبسي

الكراس = الاهتزازات والأمواج

الفوج = فيزياء (06)

الموسم الدراسي: 2015.2016

$= \frac{f_0}{2an\omega}(\sin n\pi - \sin 2n\pi + \sin n\pi)$

$a_n = 0$

$b_n = \frac{1}{4a}\int_0^{2a} f_0 \sin n\omega t . dt - \int_{2a}^{4a} f_0 \sin n\omega t . dt$

$= \frac{-1}{4an\omega}\left[f_0 \cos n\omega t \Big|_0^{2a} - f_0 \cos n\omega t \Big|_{2a}^{4a} \right]$

$= \frac{-f_0}{\quad}\left[(\cos 2a\, n\frac{\pi}{2a} - \cos a) - (\cos 4a\, n\frac{\pi}{2a} - \cos 2a\, n\frac{\pi}{2a}) \right]$

$= \frac{-f_0}{\quad}\left[(\cos n\pi - 1) - (\cos 2an\pi - \cos n\pi) \right]$

$= \frac{f_0}{\quad}\left[1 - \cos n\pi + \cos 2an\pi - \cos n\pi \right]$

$= \frac{f_0}{\quad}\left[2 - 2\cos n\pi \right]$

$b_n = \frac{f_0}{n\pi}(1 - \cos n\pi)$

$f(t) = \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{f_0}{n\pi}(1 - \cos n\pi)\sin n\omega t$

التمرين السابع =

$J = J_{\text{مركز}} + J_{\text{إضافي}} = \frac{1}{2}mR^2 + mR^2$ n

$= \frac{3}{2}mR^2$

لدينا $J\ddot{\theta} = \Sigma I$

$\frac{3}{2}mR^2\ddot{\theta} = -mgR\sin\theta$

$\frac{3}{2}mR^2\ddot{\theta} + mgR\sin\theta = 0$

2014.10.20

$f(t) = a_0 + \sum_{n=1} a_n \cos \omega t + b_n \sin \omega t$

$a_0 = \frac{1}{T}\int_0^T f(t)$

$a_n = \frac{1}{T}\int_0^T f(t)\cos n\omega t$

$b_n = \frac{1}{T}\int_0^T f(t)\sin n\omega t . dt$

السلسلة (01) =

التمرين 08 =

$a_0 = \frac{1}{4a}\left[\int_0^{2a} f_0\, dt + \int_0^{4a} -f_0\, dt \right]$

$= \frac{1}{4a}\left[f_0 2a - f_0 2a \right] = 0$

$a_n = \frac{1}{4a}\left[\int_0^{2a} f_0 \cos n\omega t . dt + \int_{2a}^{4a} -f_0 \cos n\omega t . dt \right]$

$= \frac{1}{4a}\left[\left[f_0 \frac{1}{n\omega}\sin n\omega t \right]_0^{2a} + \left[-f_0 \frac{1}{n\omega}\sin n\omega t \right]_{2a} \right]$

$= \frac{1}{4a}\left[\left[\frac{1}{n\omega} f_0 (\sin 2an\omega) \right] - \left[\frac{1}{n\omega} f_0 (\sin 4an\omega) \right] \sin 2an\omega \right]$

$= \frac{1}{4an\omega} . f_0 \left(\sin 2an\frac{\pi}{2a} - \sin 4an\frac{\pi}{2a} + \sin 2an\frac{\pi}{2a} \right)$

2014.10.26

السلسلة (2)

التمرين الأول:

الشكل (1):

1- ايجاد الطاقة الحركية:

$T = T_m + T_\mu + T_m$

$T_m = \frac{1}{2} J_m \dot{\theta}^2 \quad , \quad J_m = \frac{1}{2} m R^2$

$= \frac{1}{4} m R^2 \dot{\theta}^2$

$T_\mu = \frac{1}{2} J_\mu \dot{\theta}^2 \quad , \quad J_\mu = \mu d^2$

$= \frac{1}{2} \mu d^2 \dot{\theta}^2$

$T_m = \frac{1}{2} m \dot{x}_m^2 = \frac{1}{2} m R^2 \dot{\theta}^2$

$T = \frac{1}{4} m R^2 \dot{\theta}^2 + \frac{1}{2} \mu d^2 \dot{\theta}^2 + \frac{1}{2} m R^2 \dot{\theta}^2$

- ايجاد الطاقة الكامنة:

$U = U_m + U_\mu$

$U_m = -\int_0^{\theta} m g \, dy = -m g R \theta$

$y = d \cos(\theta_0 + \theta)$

$U_\mu = -\int \mu g \, dy = -\mu g \left[d\cos(\theta_0+\theta) - d\cos\theta_0 \right]$

$= \mu g d \left[\cos\theta_0 - \cos\theta \right]$

$= \mu g d \left(\frac{\theta^2}{2} \cos\theta_0 + \theta \sin\theta_0 \right)$

$U = -m g R \theta + \mu g d \left(\frac{\theta^2}{2} \cos\theta_0 + \theta \sin\theta_0 \right)$

2- شرط التوازن:

$\frac{\partial U}{\partial \theta} = 0 \qquad \theta = 0$

$\frac{\partial U}{\partial \theta} = -m g R + \mu g d \sin\theta_0 = 0$

بما أن الاهتزازات زوايا صغيرة يكون:

$\sin\theta \simeq \theta \quad , \quad \cos\theta = 1 - \frac{\theta^2}{2}$

$\frac{3}{2} m R^2 \ddot{\theta} + m g R \theta = 0$

وهذه هي المعادلة التفاضلية للحركة.

ملاحظة:

$\ddot{\theta} + a\theta = 0$

الحل من الشكل

$\theta(t) = A \sin(\sqrt{a}\, t + \varphi)$

$= A \sin\sqrt{a}\, t + B \cos\sqrt{a}\, t$

3- الدور:

$\omega = \sqrt{\frac{2g}{3R}} \Rightarrow$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2g}{3R}}} = \sqrt{\frac{4\pi^2 \, 3R}{2g}}$

$= \sqrt{\frac{6\pi^2 R}{g}}$

$T = \sqrt{\frac{6 \cdot 0{,}2 \cdot \pi^2}{10}} = \frac{2\pi}{10}\sqrt{3} = \frac{\pi}{5}\sqrt{3} \ s$

$\theta(0) = A \sin(\varphi) = 9$

$\dot{\theta}(0) = A \sqrt{\frac{2g}{3R}} \cos(\varphi) = 0$

$\Rightarrow \cos(\varphi) = 0 \quad , \quad \varphi = \frac{\pi}{2}$

$A \sin\frac{\pi}{2} = 9^\circ \Rightarrow A = 9^\circ$

$\theta(t) = 9 \sin\left(\sqrt{\frac{2g}{3R}}\, t + \frac{\pi}{2} \right)$

$= 9 \left(\sin\omega t \cdot \cos\frac{\pi}{2} + \cos\omega t \sin\frac{\pi}{2} \right)$

$= 9 \cos\omega t$

$= 9 \cos\sqrt{\frac{2g}{3R}}\, t \quad , \quad \dot{\theta} = 9\sqrt{\frac{2g}{3R}}$

$\sin\sqrt{\frac{2g}{3R}}\, t = -\sqrt{\frac{4}{6}} \ \omega t$

تابع التمرين 07 =

$$\dot\theta\left(\frac{T}{8}\right)=-\sqrt{\frac{54g}{R}}\sin\left(\frac{2\pi}{T}\cdot\frac{T}{8}\right)$$

$$\dot\theta\left(\frac{T}{8}\right)=\sqrt{54\frac{g}{R}}\cdot\sin\frac{\pi}{4}$$

$$=-\sqrt{54\frac{g}{R}}\cdot\frac{\sqrt2}{2}=-\sqrt{27\frac{g}{R}}\ Rad/s$$

$$V_{max}=R/2\ \dot\theta_{max}\ \frac{R}{2}\cdot 9\sqrt{\frac{2g}{3R}}$$

$$=\sqrt{\frac{27}{2}Rg}\ m/s$$

التمرين السادس =

$$\langle x\rangle=\frac{1}{T_0}\int_0^T x\,dt$$

$$=\frac{1}{T}\int_0^T A\sin(\omega t+\varphi)\,dt$$

$$=-\frac{1}{T_0}\frac{A}{\omega}\cos(\omega t+\varphi)\Big|_0^T$$

$$=-\frac{A}{2\pi}\left[\cos\left(\frac{2\pi}{T}\times T+\varphi\right)-\cos\varphi\right]$$

$$=-\frac{A}{2\pi}[\cos(2\pi+\varphi)-\cos\varphi]$$

$$\langle x\rangle=0$$

$$\langle x^2\rangle=1/T_0\int x^2\,dt$$

$$=1/T\int_0^T A^2\sin^2\alpha\,.dt$$

$$=1/T\,A^2\sin^2(\omega t+\varphi).dt$$

$$=\frac{A^2}{2T}\int_0^T\left(\frac{1-\cos(2\omega t+2\varphi)}{2}\right)dt$$

$$=\frac{A^2}{2T}\left(\int_0^T dt-\int_0^T\cos(2\omega t+2\varphi)dt\right)$$

شرط الاهتزاز:

$$\frac{\partial^2 U}{\partial\theta^2}>0$$

$$\frac{\partial^2 U}{\partial\theta^2}=Mgd\cos\theta_0>0$$

3- المعادلة التفاضلية للحركة:

$$L=T-U$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right)-\frac{\partial L}{\partial\theta}=0 \quad (1)$$

4-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right)=\frac12(mR^2+Md^2+mR^2)\ddot\theta$$

$$\frac{\partial L}{\partial\theta}=-(-mgR+Mgd(\theta\cos\theta_0+\sin\theta_0))$$

$$(1)\Leftrightarrow\left(\frac{m}{2}R^2+Md^2+mR^2\right)\ddot\theta+(-mgR+Mgd(\theta\cos\theta_0+\sin\theta_0))=0$$

وبالتالي،

$$\left(\frac{m}{2}R^2+Md^2+mR^2\right)\ddot\theta+Mgd\cos\theta_0\,\theta=0$$

$$\ddot\theta+\frac{Mgd\cos\theta_0}{\frac{m}{2}R^2+Md^2+mR^2}\theta=0$$

وهي من الشكل:

$$\ddot\theta+a\theta=0$$

معادلة الحركة:

$$\theta(t)=A\sin(\omega t+\varphi)$$

$$\omega=\sqrt{\frac{Mgd\cos\theta_0}{\frac{m}{2}R^2+Md^2+mR^2}}$$

$\varphi = 41,81 \quad \varphi' = \varphi = 138,19$

$\dot{x}(0) = 0,03 \cdot 20\pi \cos(41,81)$

$x(t) = 0,03 \cdot \sin(20\pi + 0,96\pi)$

2014.11.09

تابع = 1 =

الشكل (2):

1) - إيجاد الطاقة الحركية .

$T = T_{3m} + T_m$

$T_{3m} = \frac{1}{2} J \dot{\theta}^2$

$J = \frac{1}{12} 3mL^2 + 3m(\frac{L}{2})^2$

$J = \frac{1}{4} mL^2 + 3m \cdot \frac{L^2}{4} = mL^2$

$T_{3m} = \frac{1}{2} mL^2 \dot{\theta}^2$

$T_m = \frac{1}{2} J \dot{\theta}^2$

$J = m(\sqrt{2}L)^2 = 2mL^2$

$T_m = mL^2 \dot{\theta}^2$

$T = \frac{1}{2} mL^2 \dot{\theta}^2 + mL^2 \dot{\theta}^2$

$= \frac{3}{2}(mL^2 \dot{\theta}^2)$

- إيجاد الطاقة الكامنة :

$U = U_{3m} + U_m + U_k + U_{k'}$

$U_{3m} = -\int_0^{\sin\theta} 3mg\, dy$

$= -3mg\left(-\frac{L}{2}\sin\theta - 0\right)$

$= \frac{3}{2} mgL$

$U_m = -\int_L^{-\sqrt{2}L\sin(\frac{\pi}{4}+\theta)} mg\, dy$

$= -mg\left[-\sqrt{2}L\sin(\frac{\pi}{4}+\theta) + L\right]$

$= -mgL[-\sqrt{2}\sin(\frac{\pi}{4}+\theta) + \cos\frac{\pi}{4} \cdot \sin\theta + 1$

$= -mgL[-\cos\theta - \sin\theta + 1]$

$= \frac{A^2}{2T}(T - 0) = \frac{A^2}{2}$

التمرين الخامس :

- التفرع جمع مباشر

- التسلسل جمع مقلوب .

التمرين الرابع :

1 - التواتر الأعظمي يوافق القوة الأعظمية :

$F_{max} = m \ddot{x}_{max}$

$\ddot{x}_{max} = A\omega^2$

$3cm = 0,03\ m \quad , f = 1\ Hz$

$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi\ Rad/s$

$\ddot{x} = 0,03 \cdot 4\pi^2 = 0,12\pi^2\ m/s^2$

$F_{max} = m\ddot{x} = 10 \cdot 0,12 = 1,2\pi^2\ N$

2 - $\dot{x} = A\omega = 0,03 \cdot 2\pi$

$= 0,06\pi\ m/s$

التمرين الثالث :

$x(t) = A\sin(\omega t + \varphi)$

$A = 0,03\ m$

$f = 10\ Hz \quad \omega = 2\pi f = 20\pi\ Rad/s$

$x(t) = 0,03\sin(20\pi t + \varphi)$

$\dot{x} = 0,03 \cdot 20\pi\cos(20\pi t + \varphi)$

$\begin{cases} x(0) = 0,03\sin\varphi = 0,02 \\ \dot{x} < 0 \end{cases}$

$\sin\varphi = \frac{0,02}{0,03} =$

فمعادلة تفاضلية حلها جيبي من الشكل

$$\theta(t) = A\sin(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{-mg + KL + 2K'L}{3mL}}$$

التمرين الثالث:

1- إيجاد الطاقة الحركية:

$$T_M = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2$$

$$= \frac{1}{2} M R^2 \dot{\theta}^2 + \frac{1}{4} M R^2 \dot{\theta}^2$$

$$= \frac{3}{4} M R^2 \dot{\theta}^2$$

$$T_{m_1} = \frac{1}{2} m_1 V_{m_1}^2 = \frac{1}{2} m_1 (R\dot{\theta} - L_1\dot{\theta})^2$$

$$T_{m_2} = \frac{1}{2} m_2 (R\dot{\theta} - L_2\dot{\theta})^2$$

$$T = \frac{3}{4} M R^2 \dot{\theta}^2 + \frac{1}{2} m_1 (R\dot{\theta} - L_1\dot{\theta})^2 + \frac{1}{2} m_2 (R\dot{\theta} - L_2\dot{\theta})^2$$

$$\vec{OM_1} = \vec{OO'} + \vec{O'M_1} = -R\theta \vec{i} + L_1 \sin(\theta_0 + \theta)\vec{i'} - L_1\cos(\theta_0+\theta)\vec{j'}$$

$$\vec{V}_{m_1} = ?$$

$$\vec{V}^2_{m_1}$$

- حساب الطاقة الكامنة

$$U_{m_1} = \int_{L\cos\theta_0}^{L\cos(\theta_0+\theta)} m_1 g\, dy = -m_1 g L_1 [\cos(\theta_0 + \theta) - \cos\theta_0]$$

$$= -m_1 g L_1 [\cos\theta_0 \underbrace{\cos\theta}_{(1-\frac{\theta^2}{2})} - \sin\theta_0 \underbrace{\sin\theta}_{\theta} - \cos\theta_0]$$

$$= -mg L_1 [-\frac{\theta^2}{2}\cos\theta_0 - \theta\sin\theta_0]$$

$$= mg L_1 [\frac{\theta^2}{2}\cos\theta_0 + \theta\sin\theta_0]$$

$$= -mgL[-(1-\frac{\theta^2}{2}) - \theta + 1]$$

$$= -mgL[\frac{\theta^2}{2} - \theta]$$

$$= mgL[\theta - \frac{\theta^2}{2}]$$

$$U_K = \frac{1}{2} K (\Delta x + L\theta)^2$$

$$U_{K'} = \frac{1}{2} K' (\Delta x + \sqrt{2} L\theta)^2$$

$$U = \frac{3}{2} mgL\theta + mgL(\theta - \frac{\theta^2}{2}) + \frac{1}{2}K(\Delta x + L\theta)^2 + \frac{1}{2}K'(\Delta x + \sqrt{2}L\theta)^2$$

2- شرط التوازن:

$$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0$$

$$\frac{\partial U}{\partial \theta} = \frac{3}{2} mgL + mgL[1 - \theta] + K(\Delta x + L\theta)L + K(\Delta x + \sqrt{2}L\theta)\sqrt{2}L$$

$$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0 \Leftrightarrow \frac{3}{2} mgL + mgL + KL\Delta x + \sqrt{2}LK'\Delta x = 0$$

- شرط الاهتزاز:

$$\frac{\partial^2 U}{\partial \theta^2} > 0 \Rightarrow -mgL + KL^2 + 2K'L^2 > 0$$

3- إيجاد معادلة الحركة:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) + \frac{\partial U}{\partial \theta} = 0$$

$$= 3mL^2\ddot{\theta} + \frac{3}{2} mgL + mgL - mgL\theta + K\Delta x L + KL^2\theta + K'\Delta x\sqrt{2}L + 2K'L^2\theta = 0$$

$$3mL^2\ddot{\theta} + (-mgL + KL^2 + 2K'L^2)\theta = 0$$

$$\ddot{\theta} + \frac{-mgL + KL^2 + 2K'L^2}{3mL^2}\theta = 0$$

$+m_2 g L_2 \theta \sin\theta_0 - m_2 g L_2 \cos\theta_0 = 0$

$[\frac{3}{2}mR^2 + m_1(R-L_1)^2 + m_2(R-L_2)^2]\ddot{\theta}$

$+[m_1 g L_1 \cos\theta_0 + m_2 g L_2 \sin\theta_0]\theta = 0$

$[3m^2 + 2mR^2 + 4mR^2]\ddot{\theta} + [4mgR\cos\theta_0 + 3mgR\sin\theta_0]\theta = 0$

$9mR^2\ddot{\theta} + (4\cos\theta_0 + 3\sin\theta_0)mgR\theta = 0$

$$\ddot{\theta} + \frac{4\cos\theta_0 + 3\sin\theta_0}{9}\frac{g}{R}\theta = 0$$

$\theta(t) = A\sin(\omega t + \varphi)$

$$\omega = \sqrt{\frac{4\cos\theta_0 + 3\sin\theta_0}{9}\frac{g}{R}}$$

$\theta(0) = \pi/6 \quad \begin{cases} A\sin\varphi = \pi/6 \\ A\omega\cos\varphi = 0 \end{cases}$

$\dot{\theta}(0) = 0$

$\cos\varphi = 0$

$\varphi = (2n + \pi/2)$

$n = 0$

$\varphi = \pi/2 \qquad A = \pi/6$

2014.11.23

التمرين الخامس:

1- ايجاد المعادلة التفاضلية لحركة الشحنة

$V_L + V_C = 0$

$V_L = L\, di/dt$

$V_L = L\frac{dq^2}{dt}$ ولدينا: $i = dq/dt$

$V_C = \frac{1}{C}q$

$L\ddot{q} + q/C = 0$

$$\ddot{q} + \frac{1}{LC}q = 0$$

$U_{m_2} = \int_{L_2\sin\theta}^{L_2\sin(\theta_0+\theta)} m_2 g\, dy = -m g L_2(\sin(\theta_0+\theta) - \sin\theta_0)$

$= -m_2 g L_2[\sin\theta_0\cos\theta + \cos\theta_0\sin\theta - \sin\theta_0]$; $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\sin\theta \approx \theta$

$= -m_2 g L_2[-\frac{\theta^2}{2}\sin\theta_0 + \theta\cos\theta_0]$

$= m_2 g L_2[\frac{\theta^2}{2}\sin\theta_0 - \theta\cos\theta_0]$

$U = m_1 g L_1[\frac{\theta^2}{2}\cos\theta_0 + \theta\sin\theta_0] + m_2 g L_2[\frac{\theta^2}{2}\sin\theta_0 - \theta\sin\theta_0]$

2- ايجاد شرط التوازن:

$\frac{\partial U}{\partial \theta} = m_1 g L_1[\theta\cos\theta_0 + \sin\theta_0] + m_2 g L_2[\theta\sin\theta_0 - \cos\theta_0]$

$\left.\frac{\partial U}{\partial \theta}\right|_{\theta=0} = 0 \Leftrightarrow m_1 g L_1\sin\theta_0 - m_2 g L_2\cos\theta_0 = 0$

- ايجاد مقدار الزاوية θ_0 عند التوازن:

$m_1 g L_1\sin\theta_0 = m_2 g L_2\cos\theta_0$

$4mR\sin\theta_0 = 3mR\cos\theta_0$

$\tan\theta_0 = \frac{\sin\theta_0}{\cos\theta_0} = 3/4 \Rightarrow \theta_0 = \arctan 3/4$

$= 36{,}86^\circ$

3- شرط الاهتزاز:

$\frac{\partial^2 U}{\partial\theta^2} > 0 \Rightarrow m_1 g L_1\cos\theta_0 + m_2 g L_2\sin\theta_0$

النظام دوما يهتز

4- ايجاد معادلة الحركة:

$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) + \left(\frac{\partial U}{\partial\theta}\right) = 0 \Longrightarrow \frac{3}{2}mR^2\ddot{\theta}$

$+ m_2(R-L_2)^2\ddot{\theta}$

$+ m_1(R-L_1)^2\ddot{\theta} + m_1 g L_1\sin\theta_0$

$+ m_1 g L_1\theta\cos\theta_0 + m_1 g L_1\sin\theta_0$

$= -mg\frac{L}{3}(1-\cos\theta)$

$= -mg\frac{L}{3}(1-(1-\frac{\theta^2}{2}))$

$= -mg\frac{L}{3}(+\frac{\theta^2}{2}) = mg\frac{L}{6}\theta^2$

$U_{2m} = \int_{2L/3}^{2L/3\cos\theta} 2mg\,dy$

$= -2mg(\frac{2}{3}L\cos\theta - \frac{2}{3}L)$

$= -2mg\frac{2}{3}L(\cos\theta - 1)$

$= -\frac{4}{3}mgL(1-\frac{\theta^2}{2}-1) = \frac{2}{3}mgL\theta^2$

$U_{K_1} = \frac{1}{2}K_1(\Delta x - \frac{L}{6}\theta)^2$

$U_{K_2} = \frac{1}{2}K_2(\Delta x - \frac{L}{3}\theta)^2$

$U = mg\frac{L}{6}\theta^2 + \frac{2}{3}mgL\theta^2$

$+\frac{1}{2}K_1(\Delta x - \frac{L}{3}\theta)^2 + \frac{1}{2}K_2(\Delta x - \frac{L}{3}\theta)^2$

$L = T - U$

$= \frac{1}{2}mL^2\dot\theta^2 - (\frac{1}{2}mL\theta^2 + \frac{1}{2}K\frac{L^2}{36}\theta^2$

$+\frac{1}{2}K_2\frac{L^2}{9}\theta^2)$

$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot\theta}\right) - \frac{\delta L}{\delta\theta} = \frac{-\delta D}{\delta\dot\theta}$ / $D = \frac{1}{2}\alpha\dot x_c^2$

$x_c = \frac{L}{2}\theta \Rightarrow \dot x_c = \frac{L}{2}\dot\theta$

$D = \frac{1}{2}\alpha\frac{L^2}{2}\dot\theta^2$

$mL^2\ddot\theta + mL\theta + K\frac{L^2}{36}\theta + K_2\frac{L^2}{9}\theta$

$= \alpha\frac{L^2}{4}\dot\theta$

$mL^2\ddot\theta + \alpha\frac{L^2}{4}\dot\theta + (mgL + \frac{1}{36}K_1L^2 + \frac{1}{9}$

$K_2L^2)\theta = 0$

$K_2 = \frac{mg}{L}$, $K_1 = \frac{32mg}{L}$

$mL^2\ddot\theta + \alpha\frac{L^2}{4}\dot\theta + (mgL + \frac{1}{36}\frac{32mg}{L}$

$L^2 + \frac{1}{9}\frac{mg}{L})\theta = 0$

$\omega = \sqrt{\frac{1}{Lc}}$

$\omega = \frac{1}{\sqrt{40.10^{-3}.10^{-6}}} = 5.10^3\ \frac{Rad}{s}$

$f = \frac{1}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi} = \frac{5.10^3}{2\pi}$

$= 2,5\pi^{-1}.10^3\ Hz$

- ايجاد حل المعادلة التفاضلية :

$q(t) = A\sin(\omega t + \varphi)$

$= A\sin(5.10^3 t + \varphi)$

$i(t) = 5.10^3 A\cos(5.10^3 t + \varphi)$

عند $t = 0$

$q(0) = q_0$

$i(0) = 0$

السلسلة (03) :

التمرين الأول :

$T = T_m + T_{2m}$

$T_m = \frac{1}{2}J\dot\theta^2$ / $J = m(\frac{L}{3})^2$

$= \frac{1}{18}mL^2\dot\theta^2$

$T_{2m} = \frac{1}{2}J\dot\theta^2$ / $J = 2m\frac{4L^2}{9}$

$= \frac{4}{9}mL^2\dot\theta^2$

$T = \frac{1}{18}mL^2\dot\theta^2 + \frac{4}{9}mL^2\dot\theta^2 =$

$\frac{9}{18}mL^2\dot\theta^2 = \frac{1}{2}mL^2\dot\theta^2$

$U = U_{K_1} + U_{K_2} + U_m + U_{2m}$

$U_m = -\int_{-L/3}^{-\frac{L}{3}\cos\theta} mg\,dy = -mg(-\frac{L}{3}\cos\theta + \frac{L}{3}$

$\delta^2 \cdot 4\pi^2 = 10^{-2}(\omega^0 - \delta^2)$

$\delta^2 \cdot 4\pi^2 = 10^{-2}\omega_0 - 10^{-2}\delta^2$

$\delta^2(4\pi^2 + 10^{-2}) = 10^{-2}\omega_0$

$\frac{\alpha^2}{64m^2}(4\pi^2 + 10^{-2}) = 10^{-2} \cdot 2\frac{g}{L}$

$\frac{\alpha^2}{64m^2} 2\pi^2 = 10^{-2} \, g/_{L}$

$\alpha^2 = 10^{-2} \frac{g}{L} \cdot \frac{64m^2}{2\pi^2}$

$\alpha = \frac{10^{-1} \cdot 8 \cdot m}{\pi}\sqrt{\frac{g}{2L}} \quad \frac{kg}{}$

Edu: 11.30

$\theta(t) = C \cdot e^{-\delta t} \sin(\omega_a t + \varphi)$

$\dot\theta(t) = -\delta C e^{-\delta t} \sin(\omega_a t + \varphi)$
$+ C\omega_a e^{-\delta t} \cos(\omega_a t + \varphi)$

$\theta(0) = 0$

$\dot\theta(0) = -4$

$\theta(0) = C \sin\varphi = 0$

$\dot\theta(0) \;\; -\delta \cdot C \sin\varphi + C\omega_a \cos\varphi = -4$

$\sin\varphi = 0 \Rightarrow \varphi = n\pi$

$\varphi = \pi$ نأخذ $n = 1$

$-C\omega_a = -4$

$C = 4/\omega_a$

$\theta(t) = \frac{4}{\omega_a} e^{-\delta t} \sin(\omega_a t + \pi)$

$\theta(t) = \frac{-4}{\omega_a} e^{-\delta t} \sin \omega_a t$

$mL^2\ddot\theta + \alpha\frac{L^2}{4}\dot\theta + mgL(1 + \frac{8}{9} + \frac{1}{9})\theta = 0$

$mL^2\ddot\theta + \alpha L^2/_4 \dot\theta + 2mgL\theta = 0$

$\ddot\theta + \frac{\alpha}{4m}\dot\theta + \frac{2g}{L}\theta = 0$

$\ddot\theta + 2\delta\dot\theta + \omega_0^2\theta = 0$

$\Delta' < 0 \Rightarrow \delta^2 - \omega_0^2 < 0$

$\Rightarrow \delta^2 < \omega_0^2 \Rightarrow \frac{\alpha^2}{64m^2} < \frac{2g}{L}$

$\alpha^2 < \frac{64m^2 \cdot 2g}{L}$

$\Rightarrow \alpha < 8m\sqrt{2g/_L}$

$\theta(t) = Ce^{-\delta(t)}\sin(\sqrt{\omega_0^2 - \delta^2}\,t + \varphi)$
$= Ce^{-\delta t}\sin(\omega_a t + \varphi)$

حيث: $\omega_a = \sqrt{\omega_0^2 - \delta^2}$

- إيجاد ثابت التخامد α:

$\frac{\theta_4}{\theta_7} = e^{0,3}$

$\theta_4(t_4) = Ce^{-\delta t_4}\sin(\omega_a t_4 + \varphi)$

$\theta_7(t_7) = Ce^{-\delta t_7}\sin(\omega_a t_7 + \varphi)$
$= Ce^{-\delta(t_4 + 3T_a)}\sin(\omega_a(t_4 + 3t_a) + \varphi)$
$= Ce^{-\delta t_4} \cdot e^{-\delta 3 t_a} \sin(\omega_a t_4 + 6\pi + \varphi)$
$= Ce^{-\delta t_4} \cdot e^{-3\delta t_a}\sin(\omega_a t_4 + \varphi)$

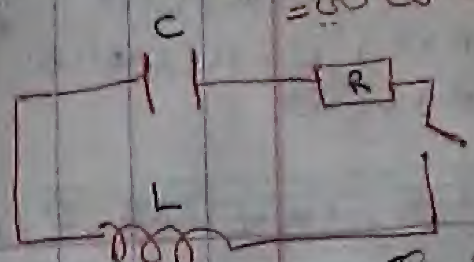
$\frac{\theta_4}{\theta_7} = \frac{1}{e^{-3\delta t_a}} = e^{3\delta t_a} = e^{0,3}$

$3\delta t_a = 0,3 \Rightarrow \delta t_a = 0,1$

$\delta \frac{2\pi}{\sqrt{\omega_0^2 - \delta^2}} = 0,1$

$\delta 2\pi = 0,1\sqrt{\omega^0 - \delta^2}$

التمرين الثاني:



أ- Q : معامل الجودة : $Q=\frac{\omega_0}{2\delta}$

$L\ddot q + R\dot q + \frac{1}{C} q = 0$

$\ddot q + \frac{R}{L}\dot q + \frac{1}{CL} q = 0$

$\ddot q + 2\delta \dot q + \omega_0^2 q = 0$ —— (1)

$2\delta = R/L$ / $\omega_0^2 = \frac{1}{CL}$

$\Delta' = \delta^2 - \omega_0^2$

$Q >> 1 \Rightarrow \omega_0 >> \delta$

$\Delta' < 0$

حل المعادلة التفاضلية (1) هو:

ج- $q(t) = A e^{-\delta t}\sin(\omega_a t + \varphi)$

حيث $\omega_a = \sqrt{\omega_0^2 - \delta^2}$

أي $\omega_a = \omega_0$

$q(t) = A e^{-\delta t} \sin(\omega_0 t + \varphi)$

لدينا: $i(t) = \frac{dq}{dt}$

$i(t) = -\delta A e^{-\delta t}\sin(\omega_0 t+\varphi)$
$+ \omega_0 A e^{-\delta t}\cos(\omega_0 t+\varphi)$
$= \omega_0 A e^{-\delta t}\cos(\omega_0 t+\varphi)$

$q(0)=q_0$

$i(0)=0$

$q(0)=A\sin\varphi = q_0$ —(*)

$i(0) = \omega_0 A\cos\varphi = 0$ —(**)

$(**) \Rightarrow \cos\varphi = 0 \Rightarrow \varphi = (2n+1)\pi/2$

نأخذ $n=0$ $\varphi = \pi/2$

$A = q_0/1$ $A = q_0$

$q(t) = q_0 e^{-\delta t}\sin(\omega_0 t + \pi/2)$

$i(t) = \omega_0 q_0 e^{-\delta t}\cos(\omega_0 t + \pi/2)$

$q(t) = q_0 e^{-\delta t}\cos\omega_0 t$

$i(t) = \omega_0 q_0 e^{-\delta t}\sin\omega_0 t$

ب) $E(t) = \frac12 L\dot q^2 + \frac12 \frac1C q^2$

$= \frac12 L\omega_0^2 q_0^2 e^{-2\delta t}\sin^2\omega_0 t$
$+ \frac{1}{2C} q_0^2 e^{-2\delta t}\cos^2\omega_0 t$ $\omega_0 = 1/LC$

$E(t) = \frac12 q_0^2 e^{-2\delta t}(L\omega_0\sin^2\omega_0 t$
$+ \frac1C\cos^2\omega_0 t)$

$E(t) = \frac{1}{2C} q_0^2 e^{-2\delta t}$

ج- $W(t) = E(0) - E(t)$

$= \frac{1}{2C}q_0^2 - \frac{1}{2C}q_0^2 e^{-2\delta t}$

استنتاج:

$W(T) = E(t) - E(t+T)$

$= \frac{1}{2C} q_0^2 e^{-2\delta t} - \frac{1}{2C} q_0^2 e^{-2\delta(t+T)}$

$W(T) = \frac{1}{2C} q_0^2 e^{-2\delta t}(1 - e^{-2\delta T})$

$= E(t)(1 - e^{-2\delta \frac{2\pi}{\omega_0}})$

$= E(t)(1 - e^{-\frac{2\pi}{Q}})$

$Q >> 1$: لدينا $x << 1$ و

بالتالي ، $e^x = 1 + x$ ⟶

$W(T) = E(t)(1 - (1 - \frac{2\pi}{Q}))$

$= \frac{3}{2} MR^2 \ddot{\theta} + m(R+L)^2 \ddot{\theta} - mgL\theta$
$+ K'(R+L)^2\theta - R(L+R)(x-(L+R)\theta)$
$= -f(R+L)^2\dot{\theta}$

$(\frac{3}{2} MR^2 + m(R+L)^2)\ddot{\theta} + f(R+L)^2\dot{\theta}$
$+(-mgL + K'(R+L)^2 + K(L+R)^2)\theta$
$= K(R+L)x$

$32 mR^2\ddot{\theta} + 16 f R^2\dot{\theta} + 3mg R\theta$
$= 4KRB \sin \Omega t$

$\ddot{\theta} + \frac{1}{2} \frac{f}{m}\dot{\theta} + \frac{3}{32}\frac{g}{R}\theta = \frac{K}{mR} B \sin \Omega t$

$\ddot{\theta} + 2\delta\dot{\theta} + \omega_0^2\theta = B' \sin \Omega t$ —(1)

$2\delta = \frac{1}{2}\frac{f}{m}$, $\omega_0^2 = \frac{3}{32}\frac{g}{R}$, $B' = \frac{1}{8}\frac{K}{mR} B$

$\theta(t) = \theta_c(t) + \theta_p(t)$

نهتم بإيجاد الحل الخاص باعتبار وجود النبض Ω في النمط الدائم

$\theta_p(t) = A \sin(\Omega t + \varphi)$

لتسهيل الحساب نستبدل في المعادلة (1) $\sin \Omega t$ بـ $e^{j(\Omega t)}$

$(1) \Rightarrow \ddot{\theta} + 2\delta\dot{\theta} + \omega_0\theta = B' e^{j\Omega t}$

$\Rightarrow \theta_p(t) = A e^{j(\Omega t+\varphi)}$

$\theta_p'(t) = j\Omega A e^{j(\Omega t+\varphi)}$

$\theta_p''(t) = -\Omega^2 A e^{j(\Omega t+\varphi)}$

بالتعويض نجد

$(-\Omega^2 + 2\delta j\Omega + \omega_0^2) A e^{j(\Omega t+\varphi)} = B' e^{j\Omega t}$

$[(\omega_0^2 - \Omega^2)A + j2\delta\Omega A] e^{j\varphi} = B'$

$(\omega_0^2 - \Omega^2)A + j2\delta\Omega A = B' e^{-j\varphi}$

$W(T) = E(t) \cdot \frac{2\pi}{\Omega}$

$=$

السلسلة رقم (04):

التمرين الأول: (1)

$T_m = \frac{1}{2} J\dot{\theta}^2 + \frac{1}{2} m V_m^2$

$= \frac{1}{4} MR^2\dot{\theta}^2 + \frac{1}{2} MR^2\dot{\theta}^2$

$= \frac{3}{4} MR^2\dot{\theta}^2$

$T_m = \frac{1}{2} m(R\dot{\theta} + L\dot{\theta})^2$

$T = \frac{3}{4} MR^2\dot{\theta}^2 + \frac{1}{2} m(L\dot{\theta} + R\dot{\theta})^2$

$U_m = -\int_{-L}^{-L\cos\theta} mg\, dy$

$= -mg(-L\cos\theta + L)$

$= -mg(\frac{\theta^2}{2})$

$U_{K'} = \frac{1}{2} K'(R\theta + L\theta)^2$

$= \frac{1}{2} K'(R+L)^2\theta^2$

$U_K = \frac{1}{2} K(x - (R+L)\theta)^2$

$U = -mg\frac{\theta^2}{2} + \frac{1}{2} K'(R+L)^2\theta^2$
$+ \frac{1}{2} K(x - (R+L)\theta)^2$

$D = \frac{1}{2} f \dot{x}_m^2$

$= \frac{1}{2} f(R\dot{\theta} + L\dot{\theta})^2$

$= \frac{1}{2} f(R+L)^2\dot{\theta}^2$

$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\theta}}\right] - \frac{\partial U}{\partial \theta} = -\frac{\partial D}{\partial \dot{\theta}}$

النظام يسير في اتجاه واحد

$\Rightarrow \ddot\theta + \frac{f}{m}\dot\theta + \frac{1}{mL^2}(mgL + \frac{1}{4}KL^2)\theta = \frac{L}{mL}F_0 \sin \Omega t$

$\Rightarrow \ddot\theta + 2\delta\dot\theta + \omega_0^2\theta = \frac{1}{L}F_0 \sin\Omega t$

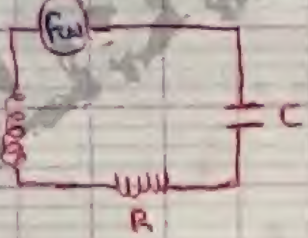
$$\ddot\theta + \frac{f}{m}\dot\theta + \frac{gm + \frac{K}{4}L}{mL}\theta = \frac{F_0}{mL}\sin\Omega t$$

$$\ddot\theta + 2\delta\dot\theta + \omega_0^2\theta = F_0'\sin\Omega t$$

$2\delta = \frac{f}{m}$, $\omega_0^2 = \frac{gm + \frac{K}{4}L}{mL}$, $F_0' = \frac{F_0}{mL}$

[illegible]

التمرين الثاني :



$L\ddot q + R\dot q + \frac{1}{C}q = E(t)$ —(1)

$\ddot q + \frac{R}{L}\dot q + \frac{1}{LC}q = \frac{E(t)}{L}$

$\ddot q + 2\delta\dot q + \omega_0^2 q = \frac{E(t)}{L}$

$2\delta = \frac{R}{L}$ $\omega_0^2 = \frac{1}{LC}$

$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = E_0 e^{j\omega t}$ —(2)

نفرض : $i = Ae^{j(\omega t+\varphi)}$

$\frac{di(t)}{dt} = j\omega A e^{j(\omega t+\varphi)}$

$\int i(t)dt = \frac{1}{j\omega}Ae^{j(\omega t+\varphi)}$

$Lj\omega Ae^{j(\omega t+\varphi)} + RAe^{j(\omega t+\varphi)} + \frac{1}{C}\frac{1}{j\omega}e^{j(\omega t+\varphi)} = E_0 e^{j\omega t}$

$(Lj\omega + R + \frac{1}{C}\frac{1}{j\omega})i(t) = E(t)$

$(R + j(L\omega - \frac{1}{C\omega}))i(t) = E(t)$

$= B'\cos\varphi - jB'\sin\varphi$

$$\begin{cases}(\omega_0^2-\Omega^2)A = B'\cos\varphi \\ 2\delta\Omega A = -B'\sin\varphi\end{cases}$$

$$A = \frac{B'}{\sqrt{(\omega_0^2-\Omega^2)^2+(2\delta\Omega)^2}}$$

$tg\varphi = \frac{-2\delta\Omega}{\omega_0^2-\Omega^2} \Rightarrow \varphi = \arctan\frac{-2\delta\Omega}{\omega_0^2-\Omega^2}$

$$\theta_p(t) = \frac{B'}{\sqrt{(\omega_0^2-\Omega^2)^2+(2\delta\Omega)^2}}\sin\left(\Omega t + \arctan\frac{-2\delta\Omega}{\omega_0^2-\Omega^2}\right)$$

(2)

$T_m = \frac{1}{2}J\dot\theta^2 = \frac{1}{2}mL^2\dot\theta^2 = T$

$U_m = -\int_L^{L\cos\theta} mg\,dy$

$= -mgL(\cos\theta - 1)$

$= -mgL(1-\frac{\theta^2}{2}-1) = mgL\frac{\theta^2}{2}$

$U_K = \frac{1}{2}Kx^2 = \frac{1}{2}K(\frac{L}{2}\theta)^2$

$= \frac{1}{2}K\frac{L^2}{4}\theta^2$

$U = mgL\frac{\theta^2}{2} + \frac{1}{8}KL^2\theta^2$

$D = \frac{1}{2}f\dot x_m^2$

$= \frac{1}{2}fL^2\dot\theta^2$

$$\frac{d}{dt}\left[\frac{\partial T}{\partial\dot\theta}\right] + \frac{\partial U}{\partial\theta} = \frac{\partial D}{\partial\dot\theta}$$

$mL^2\ddot\theta + mgL\theta + \frac{1}{4}KL^2\theta$

$= -fL^2\dot\theta + LF(t)$

$\Rightarrow mL^2\ddot\theta + fL^2\dot\theta + (mgL + \frac{1}{4}KL^2)\theta$

$= LF_0\sin\Omega t$

$$Z(\omega) = R + j\left(L\omega - \frac{1}{C\omega}\right)$$

$$|Z(\omega)|\,|i| = |E|$$

$$= \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}\; A = E_0$$

$$I = \frac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

3- الشدة الأعظمية توافق : $\left(L\omega - \frac{1}{C\omega}\right) = 0$

$$L\omega - \frac{1}{C\omega} = 0 \Rightarrow L\omega = \frac{1}{C\omega}$$

$$\omega^2 = \frac{1}{CL} \Rightarrow \omega = \sqrt{\frac{1}{CL}} = \omega_0$$

ومنه : $Z(\omega) = R$

4-

$$y = \frac{\dfrac{E_0}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}}{E_0/R} = \frac{|i|}{|i_0|}$$

$$= \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

$$= \frac{R}{\sqrt{R^2 + R^2\left(\frac{L}{R}\omega - \frac{1}{RC\omega}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{L}{R}\omega - \frac{L}{R}\cdot\frac{1}{CL}\cdot\frac{1}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{\omega_0}{2\delta}\cdot\frac{\omega}{\omega_0} - \frac{\omega_0}{2\delta}\cdot\omega_0\frac{1}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$$

5- $$= \frac{1}{\sqrt{1 + \frac{Q^2}{\omega_0^2}\left(\omega - \frac{\omega_0^2}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{Q^2}{\omega_0^2}\left(\frac{\omega^2 - \omega_0^2}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{Q^2}{\omega_0^2}\left(\frac{(\omega - \omega_0)(\omega + \omega_0)}{\omega}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + 4x^2}}$$

6- $$y = \frac{|i|}{|i_0|} = \frac{\frac{|i_0|}{\sqrt{2}}}{|i_0|} = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{1 + 4x^2}} \quad 1 + 4x^2 = 2$$

$$\Rightarrow x^2 = \frac{1}{4} \quad x = \frac{1}{2},\; x = -\frac{1}{2}$$

$$-x = -\frac{1}{2}$$

$$x = Q\,\frac{\Delta\omega}{\omega_0} = Q\,\frac{\omega_0 - \omega_2}{\omega_0} = \frac{1}{2}$$

$$\omega_2 = \frac{1}{2}\,\frac{\omega_0}{Q} + \omega_0$$

$$= \omega_0\left(1 + \frac{1}{2Q}\right)$$

$$x = \frac{1}{2}$$

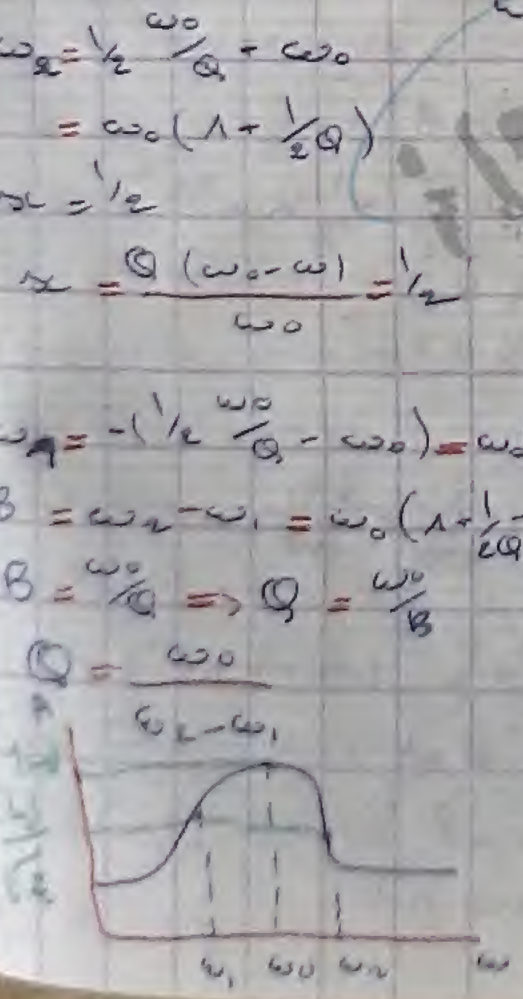
$$x = \frac{Q(\omega_0 - \omega_1)}{\omega_0} = \frac{1}{2}$$

$$\omega_1 = -\left(\frac{1}{2}\frac{\omega_0}{Q} - \omega_0\right) = \omega_0\left(1 - \frac{1}{2Q}\right)$$

7- $$B = \omega_2 - \omega_1 = \omega_0\left(1 + \frac{1}{2Q} - 1 + \frac{1}{2Q}\right)$$

$$B = \frac{\omega_0}{Q} \Rightarrow Q = \frac{\omega_0}{B}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$



السلسلة (05)

التمرين الخامس :

$$3m\ddot{x} = -T\sin\alpha + T\sin\beta$$

$$8m\ddot{y} = -T\sin\beta - T\sin\gamma$$

الاهتزاز بزوايا صغيرة جدا.

$$\sin\alpha \simeq tg\,\alpha = \frac{x}{a}$$

$$\sin\beta \simeq tg\,\beta = \frac{y-x}{a}$$

$$\sin\gamma \simeq tg\,\gamma = \frac{y}{a}$$

$$\begin{cases} 3m\ddot{x} = -T\left(\frac{x}{a}\right) + T\frac{(y-x)}{a} \\ 8m\ddot{y} = -T\frac{(y-x)}{a} - T\left(\frac{y}{a}\right) \end{cases}$$

$$3m\ddot{x} = -\frac{2}{a}Tx + \frac{T}{a}y$$

$$8m\ddot{y} = -\frac{2T}{a}y + \frac{T}{a}x$$

$$\begin{cases} 3m\ddot{x} + 12mK^2x - 6mK^2y = 0 \\ 8m\ddot{y} + 12mK^2y - 6mK^2x = 0 \end{cases}$$

$$\begin{cases} \ddot{x} + 4K^2x - 2K^2y = 0 \\ \ddot{y} + \frac{3}{2}K^2y - \frac{3}{4}K^2x = 0 \end{cases}$$

نضع : (الحلول الذاتية)

$$x = A\sin\omega t$$

$$y = B\sin\omega t$$

$$\ddot{x} = -A\omega^2\sin\omega t = -\omega^2x$$

$$\ddot{y} = -B\omega^2\sin\omega t = -\omega^2y$$

$$\begin{cases} (-A\omega^2 + 4K^2A - 2K^2B)\sin\omega t = 0 \\ (-B\omega^2 + \frac{3}{2}K^2B - \frac{3}{4}K^2A)\sin\omega t = 0 \end{cases}$$

$$A(-\omega^2 + 4K^2) - 2K^2B = 0$$

$$-\frac{3}{4}K^2A + (-\omega^2 + \frac{3}{2}K^2)B = 0$$

$$\begin{vmatrix} -\omega^2+4K^2 & -2K^2 \\ -\frac{3}{4}K^2 & -\omega^2+\frac{3}{2}K^2 \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

حتى لا يكون لدينا الحل معدوما :

$$A = B = 0$$

$$\Rightarrow \det = 0$$

$$\Rightarrow [(-\omega^2+4K^2)(-\omega^2+\frac{3}{2}K^2)] - [(-2K^2)(-\frac{3}{4}K^2)] = 0$$

$$\omega^4 - \frac{11}{2}K^2\omega^2 + \frac{9}{2}K^4 = 0$$

نضع : $\omega^2 = Z$

$$Z^2 - \frac{11}{2}K^2Z + \frac{9}{2}K^4 = 0$$

$$\Delta = \left(-\frac{11}{2}K^2\right)^2 - 4(1)\left(\frac{9}{2}K^4\right)$$

$$= \frac{121}{4}K^4 - 18K^4 = \frac{49}{4}K^4$$

$$\sqrt{\Delta} = \frac{7}{2}K^2$$

$$Z_1 = K^2 \qquad Z_2 = \frac{9}{2}K^2$$

$$\omega_1 = K \qquad \omega_2 = \frac{3}{\sqrt{2}}K$$

(حلول الحركة) ← الشكل الاهتزازي للنظام

$$x = A_1\sin(\omega_1t+\varphi_1) + A_2\sin(\omega_2t+\varphi_2)$$

$$y = B_1\sin(\omega_1t+\varphi_1) + B_2\sin(\omega_2t+\varphi_2)$$

أنماط الحركة :

أ) $\omega = \omega_1 = K$

$$(-K^2+4K^2)A - 2K^2B = 0$$

$$(3K^2)A - 2K^2B = 0$$

$$3K^2A = 2K^2B \Rightarrow B_1 = \frac{3}{2}A_1$$

ب) $\omega = \omega_2 = \frac{3}{\sqrt{2}}K$

$$(-\frac{9}{2}K^2+4K^2)A - 2K^2B = 0$$

$$-\frac{K^2}{2}A = 2K^2B \Rightarrow B_2 = -\frac{1}{4}A_2$$

$U_k = \frac{1}{2}K(x - L\theta)^2$

$U = Mg\frac{L}{4}\theta^2 + \frac{1}{2}K(x - L\theta)^2$

$= Mg\frac{y^2}{4L} + \frac{1}{2}K(x-y)^2$

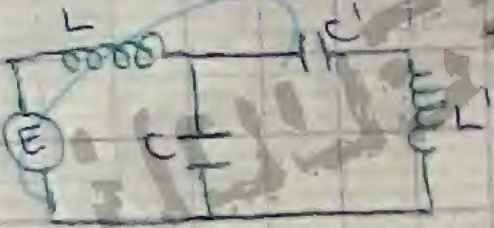
$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) + \frac{\partial U}{\partial x} = F(t)$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) + \frac{\partial U}{\partial y} = 0$

$m\ddot{x} + K(x-y) = F_0 \sin\omega t$

$\frac{1}{3}M\ddot{y} + \frac{1}{2}Mg\,y - K(x-y) = 0$

وصوا رتبا ط مزو نتي



$m\ddot{x} + Kx - Ky = F_0 \sin\omega t$

$2m\ddot{y} + 2Ky - Kx = 0$

$x = A\sin\omega t \Rightarrow \ddot{x} = -A\omega^2 \sin\omega t$

$y = B\sin\omega t \Rightarrow \ddot{y} = -B\omega^2 \sin\omega t$

$(-m\omega^2 A + KA - KB)\sin\omega t = F_0 \sin\omega t$

$(-2m\omega^2 B + 2KB - KA)\sin\omega t = 0$

$(-m\omega^2 + K)A - KB = F_0$

$-KA + 2(-m\omega^2 + K)B = 0$

$$\begin{vmatrix} -m\omega^2+K & -K \\ -K & 2(-m\omega^2+K) \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = \begin{vmatrix} F_0 \\ 0 \end{vmatrix}$$

$$\begin{vmatrix} -m\omega^2+K & -K \\ -K & 2(-m\omega^2+K) \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

ومنه :

$x = A_1\sin(\omega_1 t + \varphi_1) + A_2\sin(\omega_2 t + \varphi_2)$

$y = \frac{3}{2}A_1\sin(\omega_1 t + \varphi_1) - \frac{1}{4}A_2\sin(\omega_2 t + \varphi_2)$

-2 $\dot{x} = A_1\omega_1\cos(\omega_1 t + \varphi_1) + A_2\omega_2\cos(\omega_2 t + \varphi_2)$

$\dot{y} = \frac{3}{2}A_1\omega_1\cos(\omega_1 t + \varphi_1) - \frac{1}{4}A_2\omega_2\cos(\omega_2 t + \varphi_2)$

ومنه :

$x(0) = x_0 \Rightarrow A_1\sin\varphi_1 + A_2\sin\varphi_2 = x_0$

$y(0) = 0 \Rightarrow \frac{3}{2}A_1\sin\varphi_1 - \frac{1}{4}A_2\sin\varphi_2 = 0$

$\dot{x}(0) = 0 \Rightarrow A_1\omega_1\cos\varphi_1 + A_2\omega_2\cos\varphi_2 = 0$

$\dot{y}(0) = 0 \Rightarrow \frac{3}{2}A_1\omega_1\cos\varphi_1 - \frac{1}{4}A_2\omega_2\cos\varphi_2 = 0$

$\varphi_1 = \varphi_2 = \pi/2$

-3 $A_1 + A_2 = x_0$

$\frac{3}{2}A_1 - \frac{1}{4}A_2 = 0 \Rightarrow \frac{3}{2}A_1 = \frac{1}{4}A_2$

$\Rightarrow A_2 = 6A_1$

-4 $A_1 = x_0/7 \quad , A_2 = 6/7\, x_0$

$\varphi_1 = \varphi_2 = \pi/2$

$x = x_0/7 \sin(Kt + \pi/2) + \frac{6}{7}x_0 \sin\left(\frac{3}{\sqrt{2}}Kt + \frac{\pi}{2}\right)$

$y = \frac{3}{14}x_0\sin(Kt + \frac{\pi}{2}) - \frac{6}{18}x_0\sin\left(\frac{3}{\sqrt{2}}Kt + \pi/2\right)$

التمرين الخامس =

-1 $T = T_M + T_m$

$T_M = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\dot{\theta}^2$

$T_m = \frac{1}{2}m\dot{x}^2$

$T = \frac{1}{6}M\dot{y}^2 + \frac{1}{2}m\dot{x}^2$

-5 $U = U_m + U_k$

$U_M = -\int_{L/2}^{\frac{L}{2}\cos\theta} Mg\,dy = -Mg\frac{L}{2}(\cos\theta - 1)$

$= -Mg\frac{L}{4}\theta^2$

$\det = 0 \Rightarrow (-m\omega^2+k)2(-m\omega^2+k)$
$(k)(-k) = 0$

$\Rightarrow 2(-m\omega^2+k)^2 - k^2 = 0$

$\Rightarrow [\sqrt{2}(-m\omega^2+k)]^2 - k^2 = 0$

$\Rightarrow [-\sqrt{2}\,m\omega^2+(\sqrt{2}-1)k][-\sqrt{2}\,m\omega^2+(\sqrt{2}+1)k] = 0$

$\omega_1^2 = \dfrac{\sqrt{2}-1}{\sqrt{2}} \cdot \dfrac{k}{m} \qquad = \dfrac{\sqrt{2}+1}{\sqrt{2}} \cdot \dfrac{k}{m}$

$\begin{vmatrix} 2(-m\omega^2+k) & -k \\ -k & 2(-m\omega^2+k) \end{vmatrix} \begin{vmatrix} A \\ B \end{vmatrix} = \begin{vmatrix} F_0 \\ 0 \end{vmatrix} \quad - 6$

$A = \dfrac{2F_0(-m\omega^2+k)}{[-\sqrt{2}m\omega^2+(\sqrt{2}-1)k][-\sqrt{2}m\omega^2+(\sqrt{2}+1)k]}$

$B = \dfrac{F_0 \cdot k}{[-\sqrt{2}m\omega^2+(\sqrt{2}-1)k][-\sqrt{2}m\omega^2+(\sqrt{2}+1)k]}$